#3.3.3

> **From:** Sean Mason
> **Sent:** Saturday, April 4, 2020 2:25 PM
> **To:** Raymond Lai <Lai@camosun.bc.ca>
> **Subject:** 250B 3.3.3 Help
>
> Hi Raymond, after spending the better part of 3 hours trying to get this question I've decided to reach out for some help.
> My first action from looking through the similar examples would be the equation for flux.

$$\Phi = \iint_S \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{n}}\, dS.$$

> We already know F and so that leaves us to find the normal unit vector.

$$\vec{\boldsymbol{n}} = \frac{\vec{\boldsymbol{N}}}{\|\vec{\boldsymbol{N}}\|}$$

> We then solve for the fraction using the following:

$$\vec{\boldsymbol{N}} = \pm\nabla F = \pm\nabla[z(x,y) - z] = \pm\left[\, z_x(x,y)\,\vec{\mathbf{i}} + z_y(x,y)\,\vec{\mathbf{j}} - \vec{\mathbf{k}}\right]$$

> and $\sqrt{1 + [z_x(x,y)]^2 + \left[z_y(x,y)\right]^2}$ for ||N||.
> Solving for the numerator we get $\pm\left[\, 2x\,\vec{\mathbf{i}} + 0\,\vec{\mathbf{j}} - \vec{\mathbf{k}}\right]$
> However the notes only say that

The use of positive or negative sign depends on the orientation of the surface.

> It does not explain much more than that. **Some more info on this would be appreciated.**

The question asks for the outward flux, so we need to pick the sign (positive or negative) so that the normal vector points outward from the solid region T, hence (from the picture) $\vec{\boldsymbol{N}}$ needs to point upward which means the third component of $\vec{\boldsymbol{N}}$ (that is, the coefficient of $\vec{\mathbf{k}}$) needs to be positive and this is achieved by picking the negative sign:

$$-\left(2x\,\vec{\mathbf{i}} + 0\,\vec{\mathbf{k}} - \vec{\mathbf{k}}\right) = -2x\,\vec{\mathbf{i}} - 0\,\vec{\mathbf{k}} + \vec{\mathbf{k}}$$

> The denominator is found via the following calc.

$$\|\vec{\boldsymbol{N}}\| = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} = \sqrt{1 + (2x)^2 + 0} = \sqrt{1 + 4x^2}$$

> So we get $\vec{\boldsymbol{n}} = \frac{-2x\,\vec{\mathbf{i}} + 0\,\vec{\mathbf{j}} + \vec{\mathbf{k}}}{\sqrt{1+4x^2}}$
> However looking at the solutions

$$\iint_R \left(\left(x^2 + y(x^2)\right)\vec{\mathbf{i}} + [y(x + x^2)]\,\vec{\mathbf{j}} + \left(y^2 - x(x^2)\right)\vec{\mathbf{k}}\right) \cdot \left(-2x\,\vec{\mathbf{i}} + 0\,\vec{\mathbf{j}} + \vec{\boldsymbol{k}}\right) dA$$

> it appears as though the denominator was not used or I have missed something.

We do have the denominator when we compute the dot product "$\vec{F} \cdot \vec{n}$"; however, to set up the double integral for this surface integral, we use formula (37) and the denominator is cancelled by the "square root factor" $\sqrt{1 + [z_x(x, y)]^2 + [z_y(x, y)]^2}$ in that formula.

> The rest of the problem relies on this calculation so I am currently stuck at this point. Further, Looking at the solution it seems that you have substituted Z=x^2 in F. This is not discussed in any examples as far as I can see and I would **definitely appreciate more info on this.**

When we apply formula (37), we replace the variable z by a function of x and y representing that variable: on the right hand side of formula (37), the function $f(x, y, z)$ is replaced by $f\big(x, y, z(x, y)\big)$.
For instance, in the solution of the example on page 89, we have $f(x, y, z) = x + 2z$; when we set up the double integral, since $z = 6 - 3x - 2y$, we replace $x + 2z$ by $x + 2(6 - 3x - 2y)$.