

$$\#2.5.3. \frac{\partial(x,y,z)}{\partial(\rho,\phi,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \phi} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \phi} & \frac{\partial y}{\partial \theta} \\ \frac{\partial z}{\partial \rho} & \frac{\partial z}{\partial \phi} & \frac{\partial z}{\partial \theta} \end{vmatrix} = \dots \text{ (fill in the gap) } \dots$$

$$\begin{aligned}
&= \rho^2 \sin \phi \cos^2 \phi \cos^2 \theta + \rho^2 \sin \phi \cos^2 \phi \sin^2 \theta + \rho^2 \sin^3 \phi \cos^2 \theta + \rho^2 \sin^3 \phi \sin^2 \theta \\
&= \text{(factor by grouping, first two terms in one group and last two terms in another group)} \\
&= [\rho^2 \sin \phi \cos^2 \phi \cos^2 \theta + \rho^2 \sin \phi \cos^2 \phi \sin^2 \theta] + [\rho^2 \sin^3 \phi \cos^2 \theta + \rho^2 \sin^3 \phi \sin^2 \theta] \\
&= \dots \left(\begin{array}{l} \text{take out common factors in each group,} \\ \text{simplify using } \cos^2 \theta + \sin^2 \theta = 1 \end{array} \right) \text{(fill in the gap)} \dots \\
&= \rho^2 \sin \phi \cos^2 \phi + \rho^2 \sin^3 \phi \\
&= \dots \text{(take out common factor, simplify using } \cos^2 \phi + \sin^2 \phi = 1 \text{) (fill in the gap) } \dots \\
&= \boxed{\rho^2 \sin \phi}
\end{aligned}$$