Odd-Root Property

If n is an odd positive integer, then for any real number k,

$$x^n = k \qquad \text{is equivalent to} \qquad x = \sqrt[n]{k}$$

Even-Root Property

If n is an even positive integer, then

$$x^n = k \qquad \begin{cases} \text{is equivalent to } x = \pm\sqrt[n]{k} & \text{if } k > 0 \\ \text{is equivalent to } x = 0 & \text{if } k = 0 \\ \text{has no real solution} & \text{if } k < 0 \end{cases}$$

Raising Each Side of an Equation to a Power

If n is odd, then $a = b$ and $a^n = b^n$ are equivalent equations.

If n is even, then $a = b$ and $a^n = b^n$ may not be equivalent; however, the solution set to $a^n = b^n$ contains all of the solutions to $a = b$.

Strategy for Solving Equations with Exponents and Radicals

1. In raising each side of an equation to an even power, we create an equation that gives extraneous solutions. We must check all possible solutions in the original equation.
2. When applying the even-root property, remember that there is a positive and a negative even root for any positive real number.
3. For equations with rational exponents, raise each side to a positive or negative integral power first and then apply the even- or odd-root property.
   (Positive fraction – raise to a positive power; negative fraction – raise to a negative power.)