

Determine whether each of the following matrix is in row-echelon form. If so, is the matrix also in reduced row-echelon form? If not, which condition(s) of a row-echelon form fail(s)?

$$(a) \left[ \begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 5 \end{array} \right]$$

Answer: This matrix is not in row-echelon form; it fails the condition

(i) If a row does not consist entirely of zeros, then the first nonzero number in the row is a 1.

$$(b) \left[ \begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 6 \end{array} \right]$$

Answer: This matrix is not in row-echelon form; it fails the conditions

(i) If a row does not consist entirely of zeros, then the first nonzero number in the row is a 1,

(ii) If there are any rows that consist entirely of zeros, then they are grouped together at the bottom of the matrix.

$$(c) \left[ \begin{array}{ccc|c} 0 & 1 & 2 & 3 \\ 1 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Answer: This matrix is not in row-echelon form; it fails the condition

(iii) In any two successive rows that do not consist entirely of zeros, the leading 1 in the lower row occurs farther to the right than the leading 1 in the higher row.

$$(d) \left[ \begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 1 & 5 & 4 & 7 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Answer: This matrix is not in row-echelon form; it fails the condition

(iii) In any two successive rows that do not consist entirely of zeros, the leading 1 in the lower row occurs farther to the right than the leading 1 in the higher row.

$$(e) \left[ \begin{array}{ccc|c} 1 & -1 & 0 & 4 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

Answer: This matrix is in row-echelon form; however, it is not in reduced row-echelon form as it fails the condition:

(iv) Each column that contains a leading 1 has zeros everywhere else in that column.

$$(f) \left[ \begin{array}{cccc|c} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Answer: This matrix is in row-echelon form and reduced row-echelon form.