Probabilities

Factorial Notation: For any non-negative integer n, $n! = \begin{cases} n(n-1)(n-2)\cdots(2)(1) & \text{for } n \geq 1 \\ 1 & \text{for } n = 0 \end{cases}$.

Permutations: $P(n,r) = \frac{n!}{(n-r)!}$

Combinations: $C(n,r) = \frac{n!}{(n-r)!r!}$

Example Use the definition $C(n,k) = \frac{n!}{k!(n-k)!}$ to show that for any three non-negative integers m, r, and j satisfying $m \geq j$ and $r \geq j$, we have $C(m,r)C(r,j) = C(m,j)C(m-j,r-j)$.

Solution:

Left Hand Side = $C(m,r)C(r,j) = \frac{m!}{r!(m-r!)} \times \frac{r!}{j!(r-j!)} = \frac{m!}{j!(m-r)!(r-j)!}$

Right Hand Side = $C(m,j)C(m-j,r-j) = \frac{m!}{j!(m-j)!} \times \frac{(m-j)!}{(r-j)!\big((m-j)-(r-j)\big)!}$

$= \frac{m!(m-j)!}{j!(m-j)!(r-j)!(m-j-r+j)!} = \frac{m!}{j!(r-j)!(m-r)!}$ = Left Hand Side

Exercise

- Without using the built in function keys for combinations, permutations, or factorials on your calculator, evaluate
  - $P(8,4)$ [Answer: 1680]
  - $C(5,3) \cdot C(7,4)$ [Answer: 350]
- Given n is a positive integer, evaluate and simplify the following
  - $\left|\frac{(-1)^{n+1}2^{n+1}}{(n+1)!} \div \frac{(-1)^n 2^n}{n!}\right|$ [Answer: $\frac{2}{n+1}$]
  - $\left|\frac{(n+1)^{n+1}}{(n+1)!} \div \frac{n^n}{n!}\right|$ [Answer: $\frac{(n+1)^n}{n^n}$ or $\left(1+\frac{1}{n}\right)^n$]