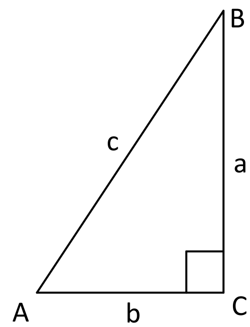
Plane Geometry

Classification of triangles:

- Scalene: No two sides are equal in length
- Isosceles: two sides are equal in length
- Equilateral: all three sides are equal in length
- Right: one angle is 90°

  Pythagorean Theorem: $a^2 + b^2 = c^2$



Solving for a variable Geometric Formulas:

- Triangle: Perimeter $P = a + b + c$, Area $A = ½\, bh$;

  Hero's formula for area of triangle:

$$A = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where} \quad s = \frac{1}{2}(a+b+c)$$

- Trapezoid: Area $A = ½\, h(b_1 + b_2)$
- Parallelogram: Area $A = bh$
- Rectangle: Area $A = LW$; Perimeter $P = 2L + 2W$
- Square (Special case of a rectangle): Apply formulae for rectangle
- Circle:

$$\text{Diameter } d = 2r, \quad \text{Area } A = \pi r^2;$$

$$\text{Perimeter (Circumference) } C = 2\pi r \text{ or } \pi d; \quad \text{Arc length } s = r\theta$$

Example Find the circumference and area of the circle with radius 10 *m*.

Solution:

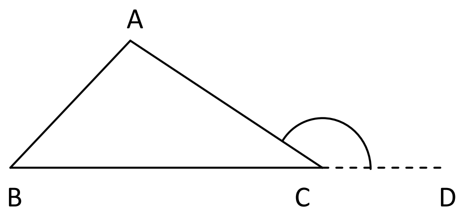
Circumference $= 2\pi r = 2\pi(10 \text{ m}) = \mathbf{20\pi \text{ m}}$

Area $= \pi r^2 = \pi(10 \text{ m})^2 = \pi(100 \text{ m}^2) = \mathbf{100\pi \text{ m}^2}$

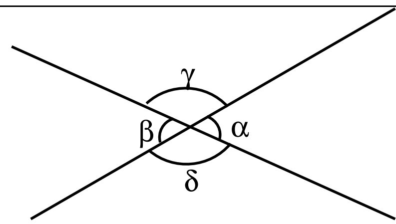
Plane Geometry

<u>Angles and Parallel Lines</u>

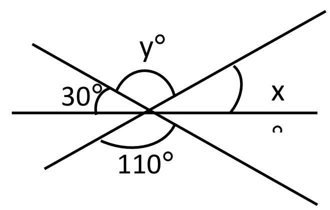
- The sum of the three interior angles of a triangle is equal to 180°.
- The sum of any two interior angles of a triangle equals the exterior angle at the remaining vertex:

	$\angle BAC + \angle ABC = \angle ACD$

- Vertically opposite angles are equal (abbreviated as "vert. opp. ∠ s")

	$\angle\alpha = \angle\beta, \quad \angle\gamma = \angle\delta$

<u>Example</u> Find the values of x and y.



Solution:

y = 110 (vert. opp. ∠s)

30° + y° + x° = 180° (adj. ∠s on a st. line)

30° + 110° + x° = 180°

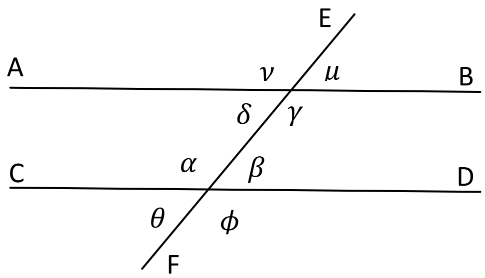
140° + x° = 180°

x° = 180° - 140°

x = **40**

Plane Geometry

- Two distinct lines are **parallel (to each other)** if they do not intersect (even if they are extended indefinitely at both ends).
- A **transversal** of two distinct lines AB and CD is a third line that cuts both AB and CD.



- We have four pair of **corresponding angles**:

$$\angle\alpha \text{ and } \angle\nu, \quad \angle\beta \text{ and } \angle\mu, \quad \angle\gamma \text{ and } \angle\varphi, \quad \angle\delta \text{ and } \angle\theta.$$

- The angles $\angle\alpha$, $\angle\beta$, $\angle\gamma$ and $\angle\delta$ are called **interior angles**; the angles $\angle\theta$, $\angle\varphi$, $\angle\mu$ and $\angle\nu$ are the **exterior angles**.
- $\angle\beta$ and $\angle\delta$ are **alternate interior angles** (and so are $\angle\alpha$ and $\angle\gamma$); $\angle\varphi$ and $\angle\nu$ are **alternate exterior angles** (and so are $\angle\theta$ and $\angle\mu$).
- Two distinct lines AB and CD are parallel if (and only if) any of the following conditions is satisfied:
  - Interior angles on the same side of the transversal are supplementary; that is, they add up to 180° (abbreviated as "int. $\angle$s of // lines"):

$$\angle\alpha + \angle\gamma = 180°, \quad \angle\beta + \angle\gamma = 180°$$

  - Any one of the four pairs of corresponding angles are equal (abbreviated as "corr. $\angle$s of // lines"):

$$\angle\alpha = \angle\nu, \qquad \angle\beta = \angle\mu, \qquad \angle\gamma = \angle\phi, \qquad or \qquad \angle\delta = \angle\theta$$

  - Any one of the two pairs of alternate interior angles is equal (abbreviated as "alt. int.$\angle$s of // lines"):

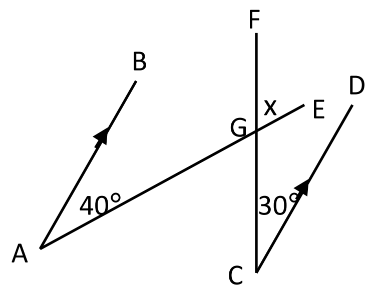
$$\angle\alpha = \angle\gamma \quad or \quad \angle\beta = \angle\delta$$

Plane Geometry

Example

In the figure, AB // CD and CF, AE meet at the point G.

Find $\angle x$.



Solution:

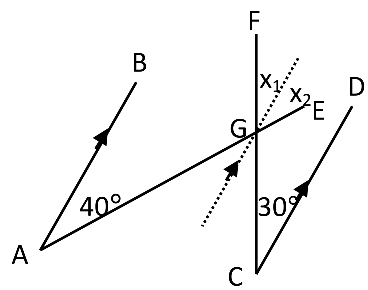
Draw a straight line GN parallel to AB and CD through G.

Let $\angle EGN = \angle x$, so that $\angle x = \angle x_1 + \angle x_2$

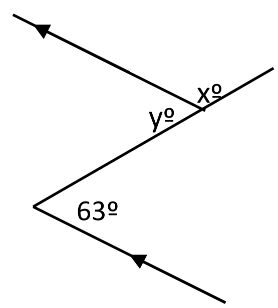
Now $\angle x_1 = 30^\circ$ (corr. $\angle$ s of // lines GN and CD)

and $\angle x_2 = 40^\circ$ (corr. $\angle$ s of // lines GN and AB)

Thus $\angle x = \angle x_1 + \angle x_2 = 30^\circ + 40^\circ = \mathbf{70^\circ}$



Example Find the values of x and y in the figure.



Solution:

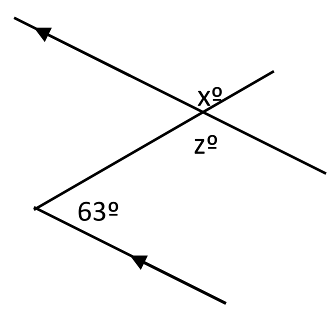
$y = \mathbf{63}$ (alt. int. $\angle$ s of // lines)

$x = 180 - y$ (adj. $\angle$s on a st. line)

$= 180 - 63$

$= \mathbf{117}$

Example Find the values of x and z in the figure.



Solution:

$z + 63 = 180$ (int. $\angle$ s of // lines)

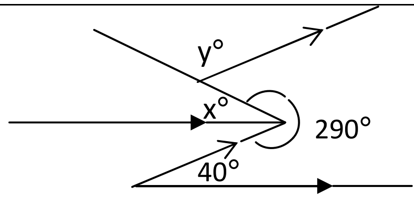
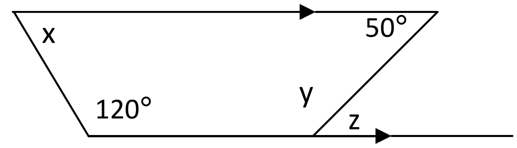
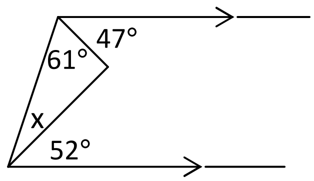
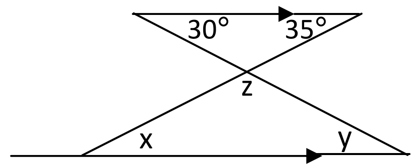
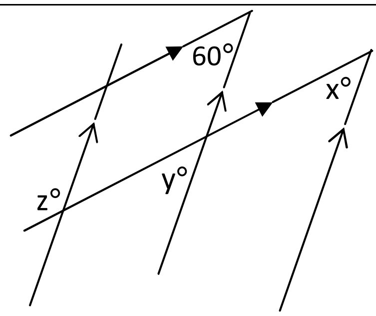
$z = 180 - 63 = \mathbf{117}$

$x = z$ (vert. opp. $\angle$s)

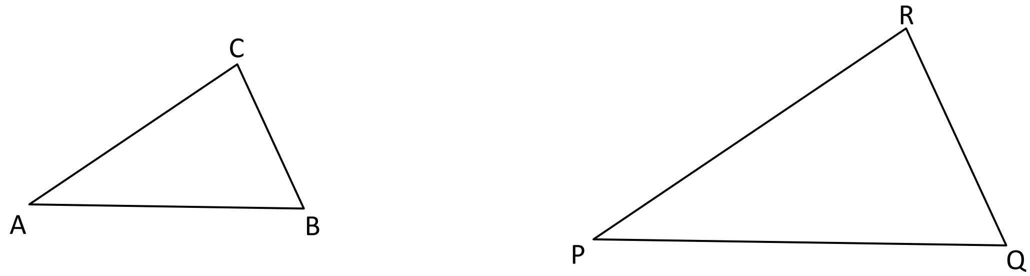
$= \mathbf{117}$

Plane Geometry

Exercises Find the unknown angles marked.

 [Answers: $x = 30, y = 110$]	 [Answers: $x = 60°, y = 130°, z = 50°$]
 [Answer: $x = 20°$]	 [Answers: $x = 35°, y = 30°, z = 115°$]
 [Answers: $x = 60, y = 60. z = 120$] Ex	

Similar Triangles



ΔABC is similar to ΔPQR, denoted by ΔABC ~ ΔPQR, if (and only if) any of the following conditions is satisfied:

- The three angles of one triangle are equal to the three angles of the other respectively (abbreviated as AAA):

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$$

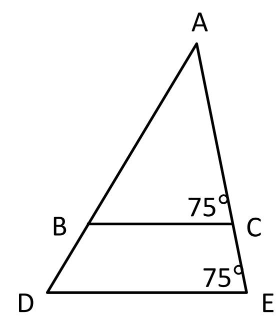
- The ratio of one side of one triangle to one side of the other triangle is equal to the ratio of another pair of sides of the two triangles, and the included angles of these sides in the triangles are equal (abbreviated as SAS): for instance,

$$\frac{AB}{PQ} = \frac{AC}{PR} \quad \text{and} \quad \angle A = \angle P$$

- The ratios of the sides of the two triangle are the same for all three sides (abbreviated as SSS):

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$$

Example Find all similar triangles in the figure.



Solution:

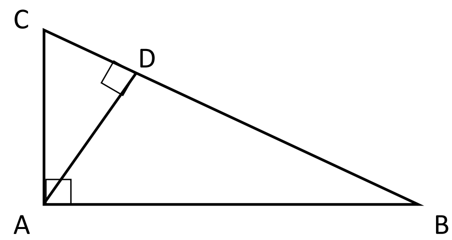
∠ACB = ∠AED = 75°

Hence BC // DE (converse of corr. ∠s of // lines)

Moreover, ∠ABC = ∠ADE (corr. ∠s of // lines)

∴ **ΔABC ~ ΔADE**

Example Find all similar triangles are there in the figure.



Solution:

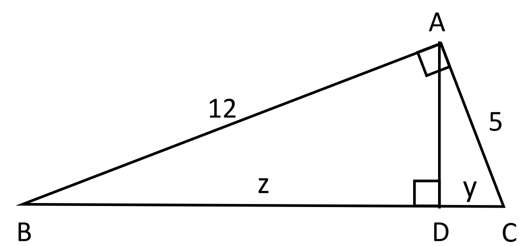
Note $\angle ADC = \angle BDA = \angle BAC = 90°$

$\angle DCA = \angle DAB = \angle ACB$

$\angle CAD = \angle ABD = \angle CBA$

Hence $\mathbf{\Delta DCA \sim \Delta DAB \sim \Delta ACB}$

Example Find y and z.



Solution:

By the Pythagorean theorem,

$BC^2 = AC^2 + AB^2$

$\Rightarrow (y+z)^2 = 25 + 144$

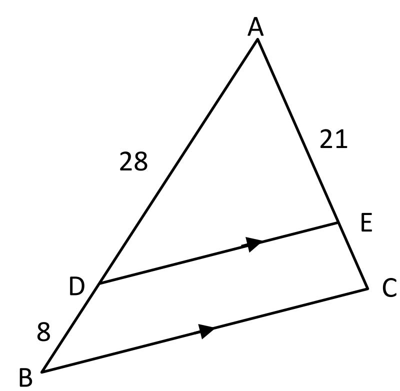
$\Rightarrow y+z = \sqrt{169} = 13$

From the previous example, $\Delta DAB \sim \Delta ACB$

Hence $\frac{BD}{AB} = \frac{AB}{CB} \Rightarrow \frac{z}{12} = \frac{12}{y+z} \Rightarrow z = \frac{12\times12}{13} = \mathbf{\frac{144}{13}}$

$y + z = 13 \Rightarrow y = 13 - \frac{144}{13} = \mathbf{\frac{25}{13}}$

Example Find EC.



Solution:

Since DE // BC, we have

$$\begin{cases} \angle ADE = \angle ABC & (\text{corr. } \angle\text{s of // lines DEand BC}) \\ \angle AED = \angle ACB & (\text{corr. } \angle\text{s of // lines DEand BC}) \\ \angle DAE = \angle BAC & (\text{Same angle}) \end{cases}$$

Hence $\Delta ADE \sim \Delta ABC$ (AAA)

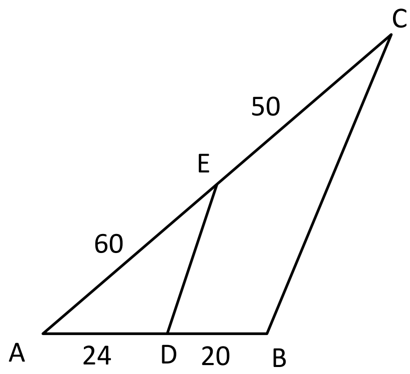
$$\frac{AB}{AD} = \frac{AC}{AE} \Rightarrow \frac{AD+DB}{AD} = \frac{AE+EC}{AE}$$

$$\Rightarrow 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE} \Rightarrow \frac{8}{28} = \frac{EC}{21}$$

$$\therefore EC = \frac{8\times21}{28} = 6$$

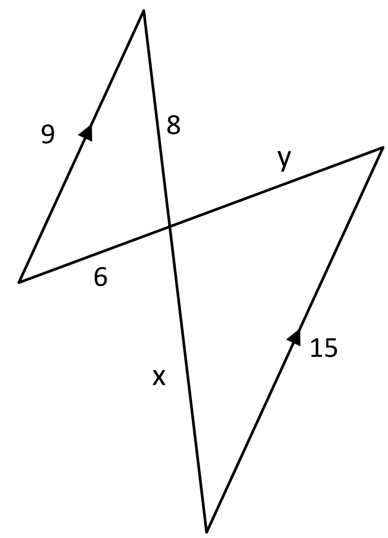
Exercises

If DE = 32, find the length of BC.



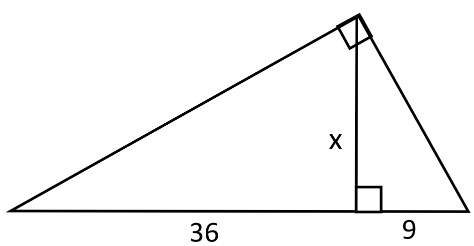
[Answer: $BC = \frac{176}{3}$]

Find x and y.



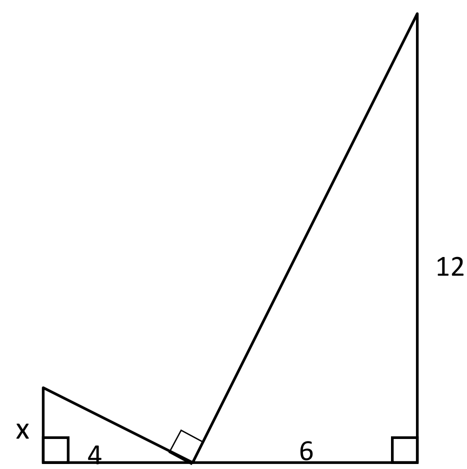
[Answers: $x = \frac{40}{3}, y = 10$]

Find x.



[Answer: $x = 18$]

Find x.



[Answer: $x = 2$]