

Binomial Theorem

Example Find the coefficient of x^5y^7 in the expansion of $(2x - 3y)^{12}$.

Solution:

(Since the power $n = 12$ is relatively large, multiplying out directly is tedious and prompt for errors; instead, we can apply the Binomial Theorem, which states that the term in the expansion involving x^5y^7 is the 8th term [$12 - r = 5 \Rightarrow r = 7$])

$$C(12,7) \times (2x)^5 \times (-3y)^7 = C(12,7)(2)^5(-3)^7x^5y^7$$

$$C(12,7)(2)^5(-3)^7 = \frac{12!}{7!5!}(2^5)[-(3^7)] = -\frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}(2^5)(3^7) = -\mathbf{55,427,328}$$

Exercise (You can use a regular scientific non-graphing non-programmable calculator)

- Find the 5th term in the expansion of $(2x - 5y)^6$. [Answer: $37,500x^2$]
- Find the 8th term in the expansion of $(3x - 2)^{10}$. [Answer: $-414,720x^3$]